Modelling the Growth of Swine Flu

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This article was originally dated 13th July, 2009. It was previously printed in the Queensland Association of Mathematics Teachers' journal (Thomson, 2009). In this publication, an articulation of the teaching and learning context has been included along with the relationship to relevant Australian curricula. A postscript has also been added reflecting on the results with the benefit of hindsight.

The swine flu pandemic

The spread of swine flu has been a cause of great concern globally. With no vaccine developed as yet, and given the fact that modern-day humans can travel speedily across the world, there are fears that this disease may spread out of control. The worst-case scenario would be one of unfettered exponential growth. Medical scientists and politicians, however, are making every effort to make this exponential growth "change its mind" and come under control. The purpose of this article is not to lay claim to a definitive mathematical model for swine flu, but rather to illustrate some activities that will help students engage with this real world phenomenon in a spirit of curiosity and exploration.

The mathematical ideas and the technology tools required to carry out an investigation of this nature are accessible to senior secondary students of calculus. The logistic function referred to in this article may not be one that is a routine part of the curriculum in every state but teachers will be able to help senior students make sense of it.

A classroom simulation

The process of encouraging exponential growth to "change its mind" can be simulated in the classroom with the aid of some random numbers and some intrepid students (Texas, 2009). If we assume that there are 25 students in the class, then each student can be assigned a number from 1 to 25. Some form of random number generator can then be used to select the first student to contract the disease. There are various ways for different calculators to do this. For example, on the Casio FX-9860G graphics calculator this can be achieved using the command INT(25*RAN#) +1 or on a device such as the ClassPad by the command RAND(1,25). The first student to be infected can then select a random number to infect someone else in the class. The next "day" these two students can then each select random numbers to infect another two students. The process continues and the whole school would be at risk if not for the courage of the teacher who dashes out the door and locks the students in. This noble act of heroism transforms an uncontrollable pandemic into an epidemic: it persuades exponential growth to "change its mind". As each "day" passes, the number of students who are infected may increase but the chances of a student being selected who has already been infected also increases. The total number of infected students then levels off to a limiting value (the number of students in the class).

This simulation was carried out at Ormiston College in south Brisbane in June 2009 with my class of fourteen Year 12 students. The sample results shown below were obtained by one of the students after he had averaged five runs of the simulation.

Day	1	2	3	4	5	6	7	8	9	10	11	12
Total number infected	1	2	3.4	5.2	8	10.5	11.75	13	13.5	13.75	14	14

Figure 1: A student's table of results from the simulation

He then used an Excel spreadsheet to produce the scatterplot shown below.

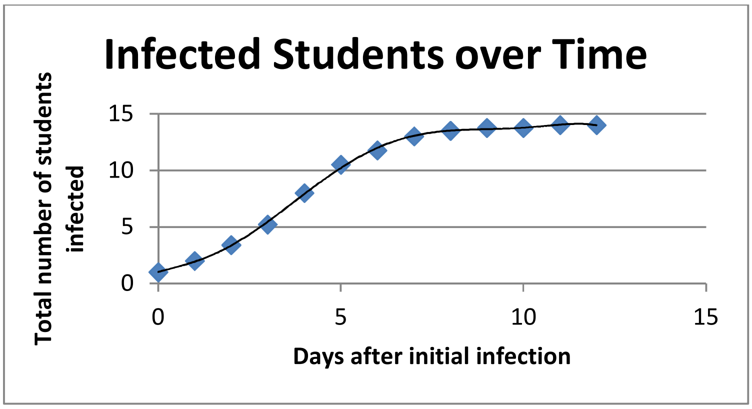


Figure 2: A student produces a scatterplot of results from the simulation

The class went on to carry out an investigation into the spread of swine flu using the logistic model for growth as described below. The students each submitted a three part report. In part (a) they presented their results from the simulation. In part (b) they used a computer algebra system to explore the application of the logistic model for growth and verified their results by manually applying their skills in calculus and algebra. Part (c) entailed researching an aspect of the growth of swine flu. Each student negotiated an individual focus for their research with me. For example, one student compared the growth of swine flu with the growth of other flu epidemics. Another student compared the mortality rates from swine flu between rich and poor countries. Other examples of areas of research included, growth in relation to population density, swine flu in large cities, and hospital admissions.

The students were working on a unit of work on Advanced Exponential Functions as part of the Mathematics C course of the Queensland Studies Authorities syllabus. The unit was designed in line with the Dimensions of Learning framework (Marzano, 1992) and had an emphasis on using knowledge meaningfully through investigation. Their reports formed part of their summative assessment for the Mathematics C course. The teaching and learning context could be related to the curriculum in other states. Relevant VCE units, for example, would be:

Mathematics Methods (CAS) – Functions and Graphs; Rates of Change and Calculus, Algebra

Specialist Mathematics – Functions, Relations and Graphs; Algebra and Calculus (particularly rates of change, point of inflection, using "solve" function on CAS calculator and second derivative).

Applying the logistic model

The logistic model has been used to describe many phenomena, for example, the growth of a population where constraints such as food supply curtail growth (Smith, 2008). The general form of the logistic model is $y = \frac{C}{1+ae^{-bx}}$, where *C* is the limiting value. Using the regression calculating power of the ClassPad, a logistic model for the spread of the disease through the class was calculated by a student to be $y = \frac{14.247}{1+13.866e^{-0.83877x}}$, where *x* is the number of days and *y* is the total number of people infected. Other devices such as the Casio FX-9860G can also be used to model the logistic function (Casio, 2010).

If we are satisfied that this is a reasonable match for our classroom simulation, can we dare to use our knowledge meaningfully and attempt to model the spread of swine flu throughout the world? The students charted the growth in the total number of people throughout the world who had been infected with swine flu since April 23rd, 2009 as illustrated in figure 3 on the right. Is there some hope that the efforts of the world's scientists and politicians have had some effect? Could the data fit a logistic model?

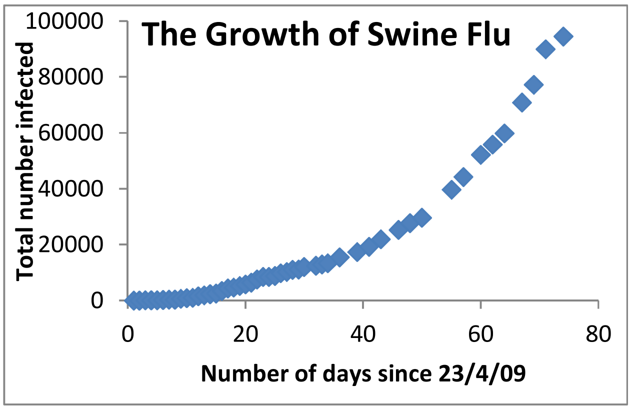


Figure 3: Growth of swine flu *(WHO, 2009)*

Using the calculator to find the logistic model for swine flu based on this data, produces $y = \frac{195315}{1+123.77e^{-0.0635x}}$, where *x* is the number of days since April 23rd and *y* is the total number of people infected. This would suggest that the total number of people infected would not rise above two hundred thousand world-wide. Whilst this might be far fewer than feared, we must remember that there are assumptions and limitations to any mathematical model. The onset of the flu season, for example, has not been considered. There are lessons from history too. It is sobering to note that the influenza pandemic of 1918 appeared to abate before returning ever more vigorously.

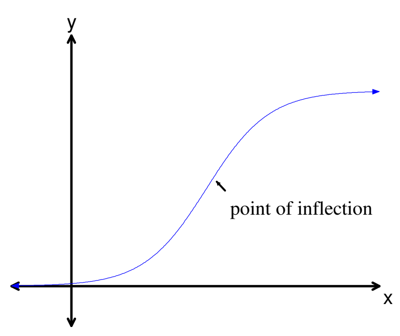
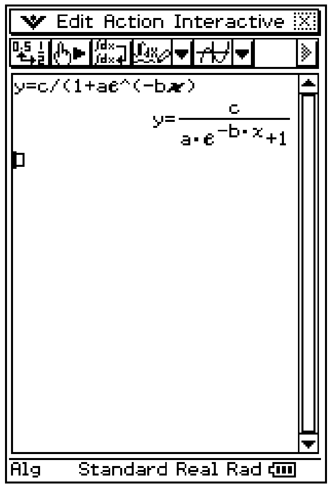


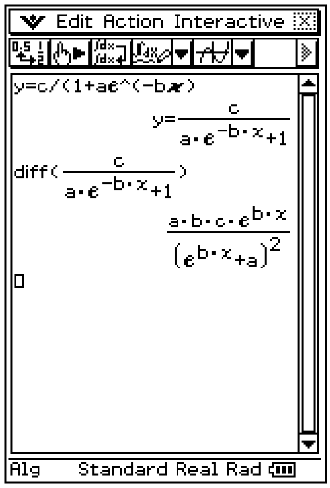
Figure 4: the point of inflection

Proceeding with caution, therefore, if we are to assume that the growth rate of the spread of swine flu has peaked, when did this occur? When might we have persuaded the exponential growth of swine flu to "change its mind"? With reference to the graph of the logistic model, this would be the point where the graph changes shape, otherwise known as the point of inflection (no pun intended). At the point of inflection, the growth rate, represented by the first derivative of the logistic function is at a maximum. Hence, at the point of inflection, the second derivative will equal zero.

A general result using computer algebra

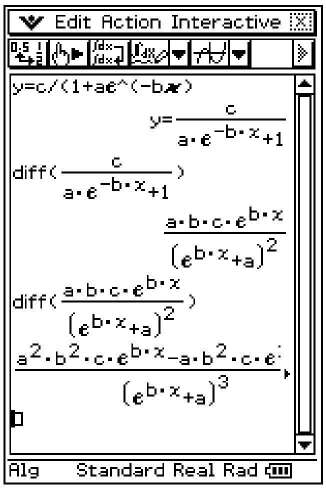


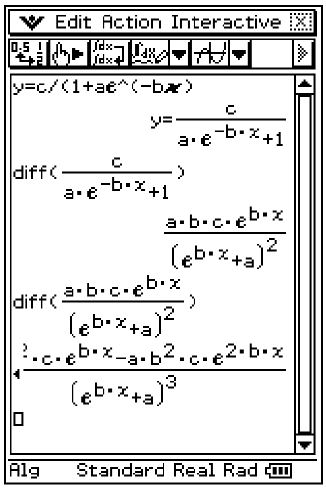
If we take an algebraic approach to finding the point of inflection, we could begin with the general logistic model. A computer algebra system (CAS) can ease the burden of manipulating complicated expressions thereby allowing students to focus more on the ideas involved rather that the manipulations. Using the computer algebra capabilities of the ClassPad, we could enter the general model $y=\dfrac{C}{1+ae^{-bx}}$ into the ClassPad as shown on the right.



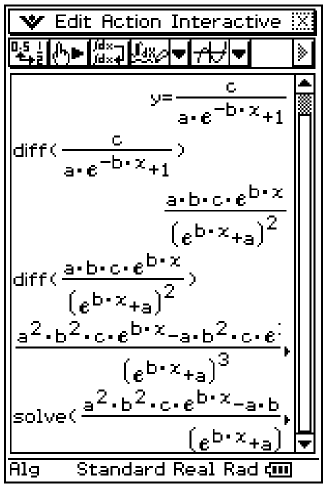
Using the ClassPad to find the first derivative we obtain the following:

Similarly, the second derivative can be found (we have to scroll across to see all of it)





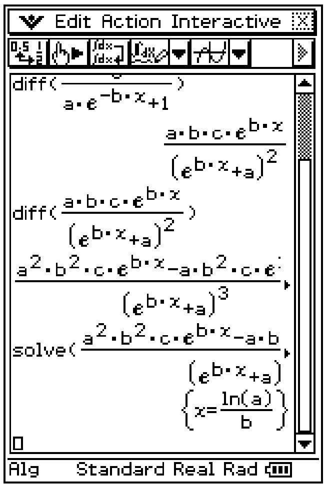
Now all we have to do is solve the equation $\dfrac{a^2b^2ce^{bx}-ab^2ce^{2bx}}{(e^{bx}+a^3)}=0$



Using the ClassPad we enter:

solve($\dfrac{a^2b^2ce^{bx}-ab^2ce^{2bx}}{(e^{bx}+a^3)}=0$)

Note: To save typing, most of this can be dragged in from the previous line using the stylus



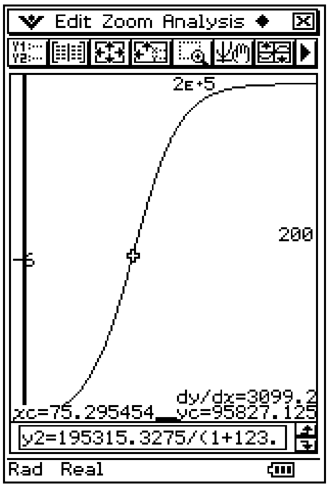
And obtain the elegantly simple result, $x=\dfrac{\ln a}{b}$

Students found that a by- hand solution to this equation could also easily be found by factorizing as shown below:

$$\frac{a^2b^2ce^{bx}-ab^2ce^{2bx}}{(e^{bx}+a^3)}=0$$
$$\therefore a^2b^2ce^{bx}-ab^2ce^{2bx}=0$$
$$\therefore ab^2ce^{bx}(a-e^{bx})=0$$
$$\therefore (a-e^{bx})=0$$
$$\therefore e^{bx}=a$$
$$\therefore bx=\ln a$$
$$\therefore x=\frac{\ln a}{b}$$

Applying this result to the swine flu data gives, $x=\dfrac{123.77}{\ln 0.0635}\approx 75.9$

To check this “black box” result we could perform the algebra by hand. Alternatively, we could check the result graphically using the graph of our swine flu model. As we trace along the graph, we can see that the growth rate, represented by *dy/dx,* reaches a maximum when *x* is approximately equal to 75.3.



All of this suggests that the growth of swine flu peaked around the 76th day after April 23rd, 2009 i.e. July 8th, 2009. At best this is an optimistic result, given the aforementioned assumptions and limitations. Despite the difficulties in obtaining data and the inevitable roughness of models of these kinds, the relatively unsophisticated modeling techniques and tools accessible to senior secondary students can produce surprisingly effective results.

Postscript

Dr Paul Armstrong from New South Wales Health was quoted on the ABC News on 20th August 2009 as saying that, “The virus seemed to have peaked its activity around mid-July” (ABC, 2009). This statement agrees with the findings of the investigation.

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