KÜRSCHÁK'S TILE

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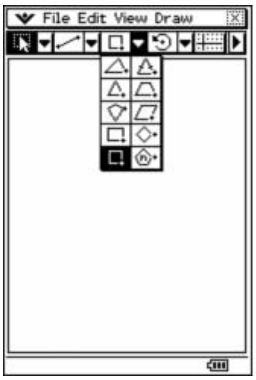
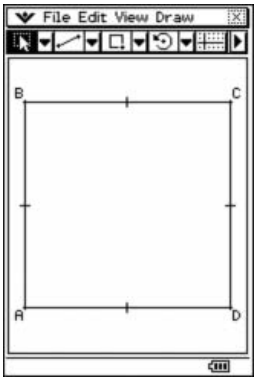
József Kürschák was a Hungarian mathematician who lived from 1864 to 1933. He made significant contributions in mathematics in the areas of geometry, calculus and linear algebra. His most important work was on the "theory of valuations". Kürschák was also a gifted teacher.

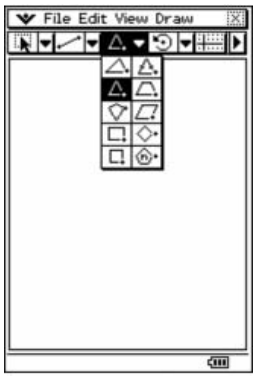
Using geometric methods, Kürschák proved that a regular dodecagon inscribed in a unit circle has an area of three square units. The geometric construction that he used in his proof is now referred to as Kürschák's tile.

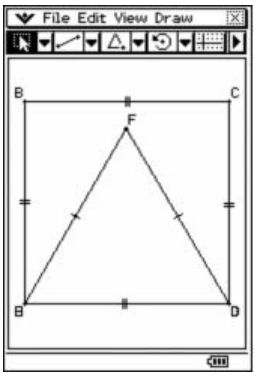
Kürschák's tile can be constructed on a ClassPad calculator by first constructing a square.



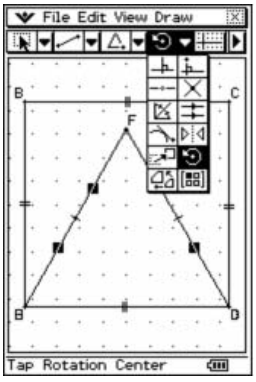


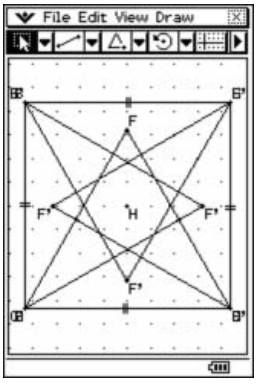
Equilateral triangles are then constructed inwards from each side of the square

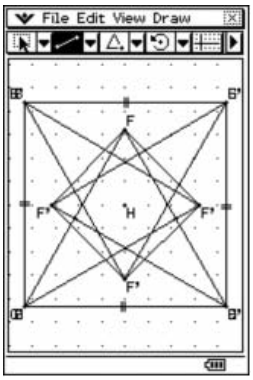




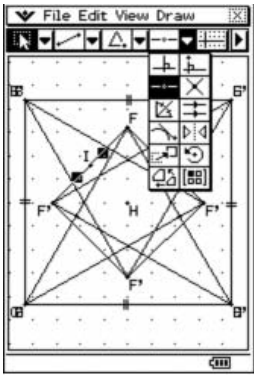
The first equilateral triangle can be rotated to generate the other three. (This is more easily achieved if the Integer Grid is turned on from the View Menu).

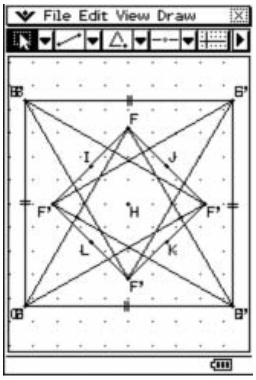




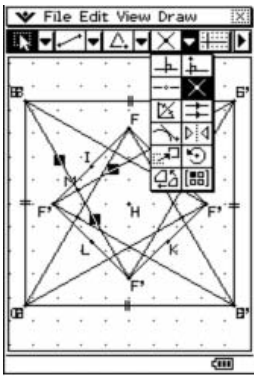


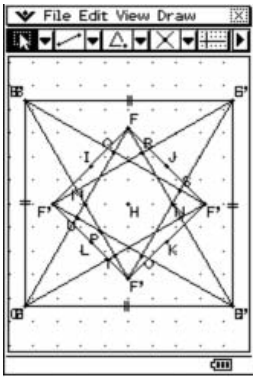
A square is then drawn by connecting the free vertices of the triangles.



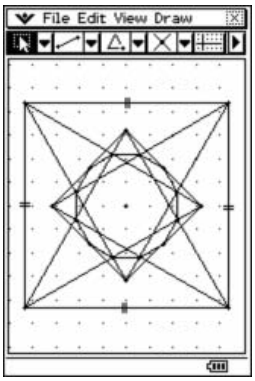


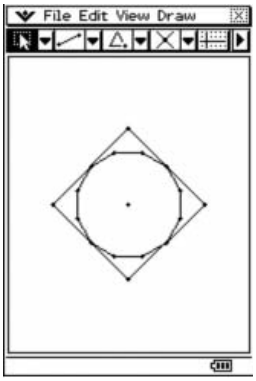
The mid-points of the sides of the square are then constructed.





The points where the triangles meet are constructed.

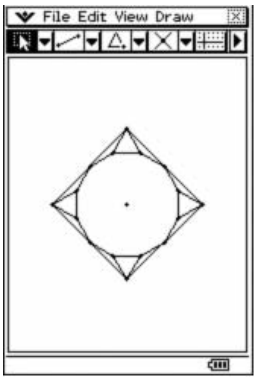


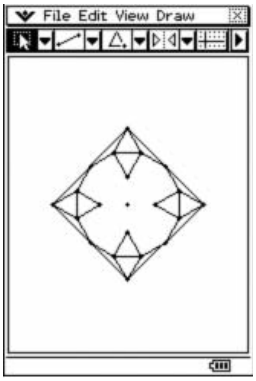


A dodecagon is then drawn using the mid-points of the square and the points where the triangles meet.

All the labels are hidden.

The original square and the four equilateral triangles are then deleted.



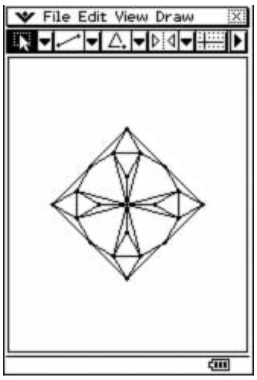


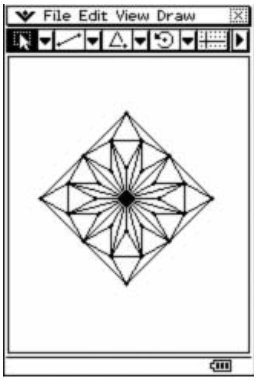
Four equilateral triangles are drawn from the dodecagon to the four vertices of the square.

Copies of these equilateral triangles are produced inside the decagon using the reflection tool.

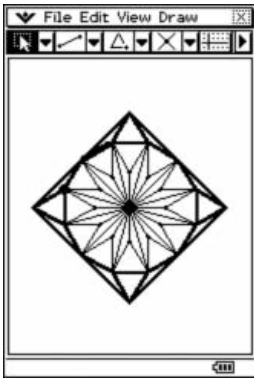
Lines are drawn from the centre to the vertices of the inner equilateral triangles.

Copies of the isosceles triangles inside the dodecagon are produced by selecting them and then rotating them first by 30 degrees and then by 60 degrees.

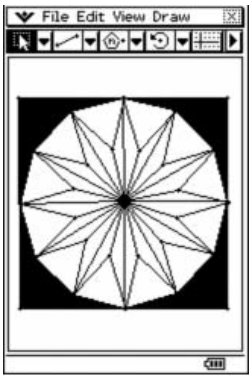


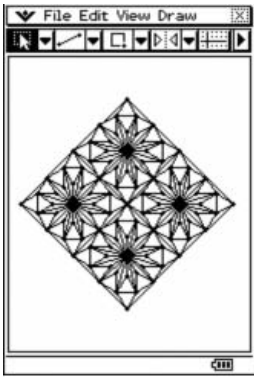


By examining Kürschák's tile, it can be seen that there are four congruent equilateral triangles and eight congruent isosceles triangles outside of the dodecagon. These twelve triangles make up one quarter of the square. Hence, if the dodecagon is inscribed in a circle of radius one unit, the square will have an area of four square units and the dodecagon will have an area of three square units.



Finally, we can also see that Kürschák's tile is aesthetically appealing, and can be used to produce attractive designs.





References

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