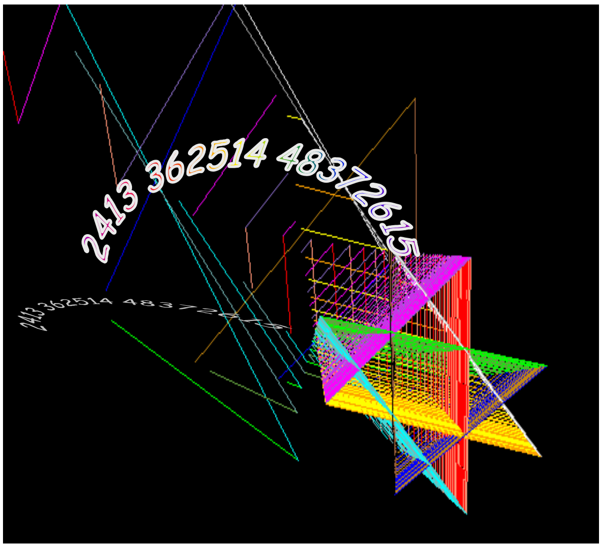
Using Technology to Transform Number Patterns into Geometric Designs



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Many of the number patterns we come across in mathematics can be transformed into fascinating geometric designs. The "nearest neighbour" number pattern. (Harradine. 1999, p.3) is an interesting example. This pattern is related to a story about a housing development on the moon in the year 3000. In the house numbering system no house number on 1st Street can be next to its first nearest neighbour (upper or lower). On 2nd Street no house number can be next to either its first or second nearest neighbour and so on.

Keeping to the minimum number of houses in each street, the pattern could be expressed as below:

1st Street   2 4 1 3

2nd Street   3 6 2 5 1 4

3rd Street   4 8 3 7 2 6 1 5

and for n^{th} Street the pattern could be generalised to:

$x \quad 2x \quad x-1 \quad 2x-1 \quad x-2 \quad 2x-2 \ldots\ldots x-n \quad 2x-n$, where $x=n+1$

A spreadsheet can be used effectively to explore this pattern and to display other patterns which emerge from it. The patterns which become evident include the natural numbers, odd numbers and even numbers. “Knight moves” produce similar results along with other patterns such as numbers which differ by 3.

2	4	1	3																				
3	6	2	5	1	4																		
4	8	3	7	2	6	1	5																
5	10	4	9	3	8	2	7	1	6														
6	12	5	11	4	10	3	9	2	8	1	7												
7	14	6	13	5	12	4	11	3	10	2	9	1	8										
8	16	7	15	6	14	5	13	4	12	3	11	2	10	1	9								
9	18	8	17	7	16	6	15	5	14	4	13	3	12	2	11	1	10						
10	20	9	19	8	18	7	17	6	16	5	15	4	14	3	13	2	12	1	11				
11	22	10	21	9	20	8	19	7	18	6	17	5	16	4	15	3	14	2	13	1	12		
12	24	11	23	10	22	9	21	8	20	7	19	6	18	5	17	4	16	3	15	2	14	1	13

Figure 1: Sequences from the Nearest Neighbour Number Pattern

In addition to the number patterns shown by the spreadsheet, a visual pattern can also be created by using conditional formatting. Having arranged the numbers in a triangle, colouring the cells yellow if they contain odd numbers and red if they contain even numbers creates the pattern shown below.

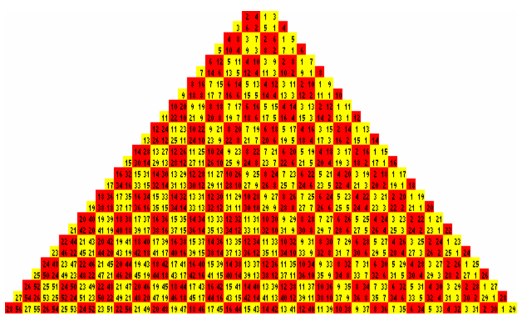


Figure 2: Conditional Formatting of Cells Creates a Visual Pattern

The idea of creating visual patterns from the number pattern can be extended further by creating spirolaterals. A spirolateral (Wells. 1991, p.239) is created by drawing a sequence of lines and turning through a constant angle between each line. The spirolateral shown below can be created from (90^0: 1,2,3).

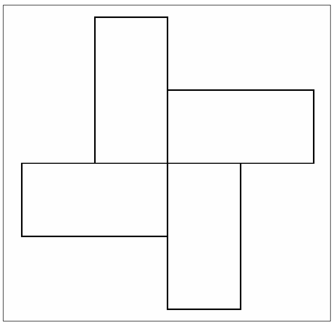


Figure 3: A Spirolateral

Using MSWLogo, the "nearest neighbour" pattern could be transformed into a spirolateral. For example, to draw 2nd Street using an angle of 60 degrees, the program would be:

FORWARD 3

RIGHT 60

FORWARD 6

RIGHT 60

FORWARD 2

RIGHT 60

FORWARD 5

RIGHT 60

FORWARD 1

RIGHT 60

FORWARD 4

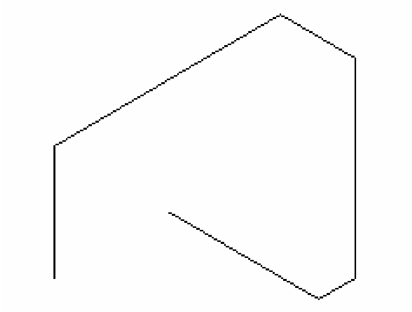


Figure 4: 2^{nd} Street 60^0

A more general program for n^{th} Street could be given by:

```
to neighbour :n :a :c
if :c>:n [stop]
FORWARD :n+1-:c
RIGHT :a
FORWARD  2*(:n+1)-:c
RIGHT :a
neighbour :n :a :c+1
```

This short program uses recursion to draw a “neighbour” based on the numbers of the houses on nth street with :a being the angle turned through before each new line is drawn. :c should be given the value 0 to begin with and then it will be allowed to count up to :n.

The angle used in the creation of a spirolateral can make a significant difference. Angles of 60^0 and 45^0 respectively were used to create figures 5 and 6 below.

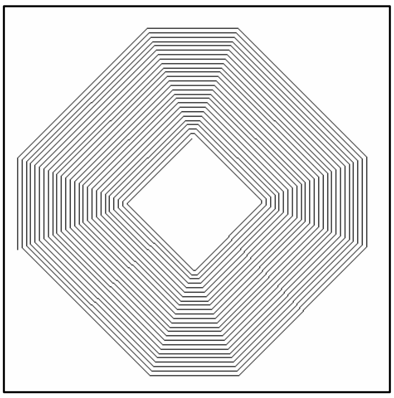


Figure 5: A “Neighbour” using 60^0

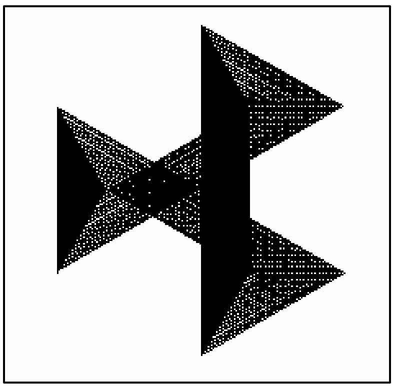


Figure 6: A “Neighbour” using 45^0

Many of the drawings produced, however, may seem to stop just when they are becoming interesting. A more elaborate version of the program would allow the user to continue drawing lines more than *n* times. This would result in negative house numbers (perhaps belonging to the dark side of the moon!) but the designs would be more intriguing.

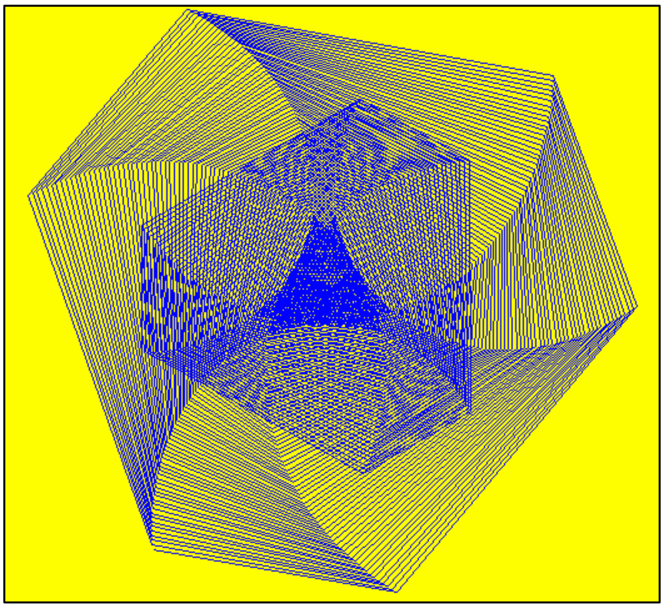
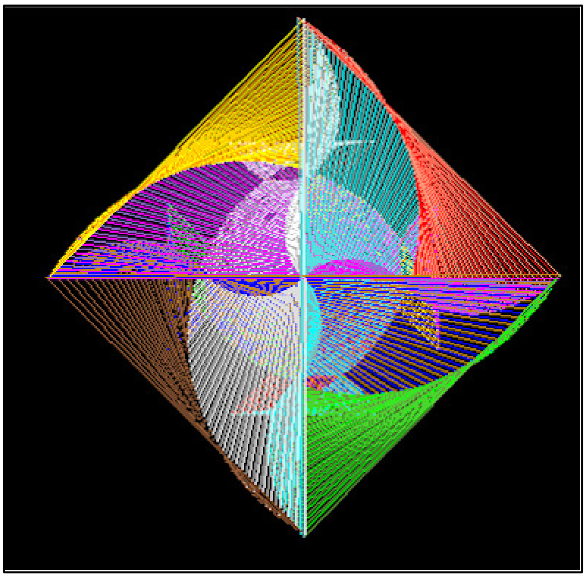


Figure 7: More than *n* lines

Several things should be considered then when running the program. How long should the first line be? This will determine the overall size of the design. What angle should be turned through between drawing each line? A tiny change in the angle can significantly affect the outcome of the design. How many lines should be drawn?



Sometimes stopping after *n* lines have been drawn is effective. Sometimes however drawing more than *n* lines gives the effect of the design being folded over into itself and some self-similarity is evident.

Experimenting with different angles and sizes can produce some impressive

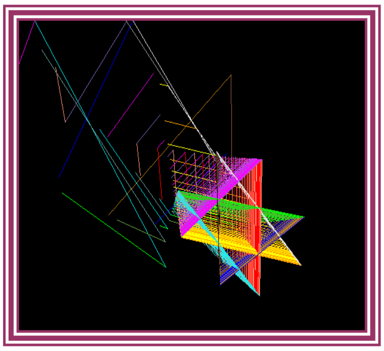
Figure 8: "Phoenix" an example of Neighbour "Artwork"

“artwork”. Some examples are shown in the Neighbour “Art Gallery” below.

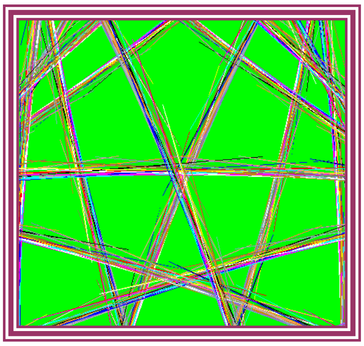
MSWLogo also allows for a multimedia effect to be created. By issuing controls to midifiles or audio files the program can set the construction of the designs to music.

“The Neighbour Art Gallery”

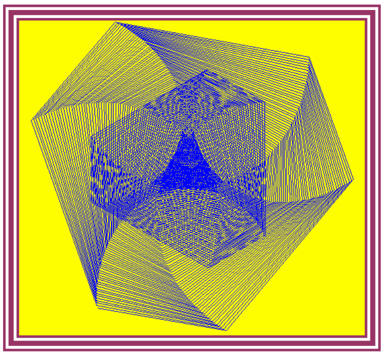
Ferris Whirl



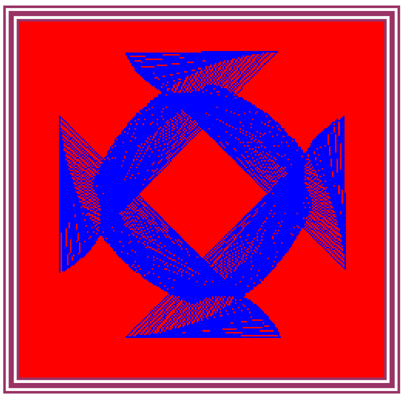
Falling Star



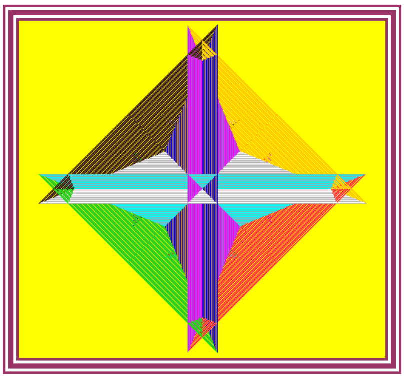
Comb



Jewel



Celtic Knot



The Ascension

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