

Foolin' around with Fibonacci

Background

Fibonacci numbers are named after a famous Italian mathematician called Leonardo Fibonacci (c. 1170-1250) sometimes known as Leonardo of Pisa.

The first few Fibonacci numbers are 1, 1, 2, 3, 5, 8, 13...

Notice that each new Fibonacci number is the sum of the two previous Fibonacci numbers.

Using Excel to produce Fibonacci numbers

Follow these steps to make the first 30 Fibonacci numbers.

- Begin by entering the heading **n** in cell **A1** and **Fn** in cell **B1** (note here that **n** represents the position of the Fibonacci number, 1st, 2nd, 3rd and so on, and **Fn** represents the corresponding Fibonacci number).
- Enter the numbers **1, 2** and **3** in cells **A2, A3, A4**
- Enter the first two Fibonacci numbers, **1** and **1** in cells **B2** and **B3**
- Now enter the formula **=B2+B3** in cell **B4**. This will calculate the third Fibonacci number.

Your spreadsheet should now look like this.

	A	B
1	n	Fn
2	1	1
3	2	1
4	3	2

Excel will now be able to help you "fill down" this pattern. Follow these steps to make it automatically generate the sequence.

- Highlight cells **A4** and **B4**.
- Roll the mouse to the bottom right corner of cell **B4** until you see some black cross hairs.
- Click and drag down to row 31 and Excel will have generated the rest of the numbers that you need.

	A	B
1	n	Fn
2	1	1
3	2	1
4	3	2

Now that you have some Fibonacci numbers in your spreadsheet, you can begin to do some investigations.

Odds and Evens

1. Find a pattern for the odd and even numbers in the Fibonacci sequence.
2. Explain why this pattern will always work.

3. Use your Excel spreadsheet to make running totals of the Fibonacci numbers in the odd positions i.e. the $1^{st}+3^{rd}+5^{th}+7^{th} \dots$, and the of those in the even position i.e. the $2^{nd}+4^{th}+6^{th}+8^{th} \dots$

You can do this by adding formulas into the next column of your spreadsheet. Follow these steps to make the running totals:

- First type in the first two Fibonacci numbers **1**, and **1**, into cells **C2** and **C3**.

- Type in the formula **=B4+C2** into cell **C4**. This will calculate the sum of the Fibonacci numbers in the 1^{st} and 3^{rd} positions.

	A	B	C
1	n	Fn	
2	1	1	1
3	2	1	1
4	3	2	3

- If you "fill down" the formula in cell **C4**, you will be able to generate the running totals of the Fibonacci numbers in the odd positions and the running totals of the Fibonacci numbers in the even positions.

It may help you to see the patterns if you lightly shade every other row. You can do this by following these steps:

- Select all the cells you have used
 - On the **Home** tab, in the **Styles** group, select **Format as Table** and chose a table format that has every other row lightly shaded.
4. What pattern do you see in the running totals of the Fibonacci numbers in the odd positions?
 5. What pattern do you see in the running totals of the Fibonacci numbers in the even positions?

Multiples

If you look at your list of Fibonacci numbers you can see the following pattern:

- Every 4th Fibonacci number is a multiple of 3
 - Every 5th Fibonacci number is a multiple of 5
 - Every 6th Fibonacci number is a multiple of 8
1. If this pattern continues, then what should every 13th Fibonacci number be a multiple of?
 2. Check your answer using the 26th Fibonacci number.

Multiplying

Try multiplying each Fibonacci number by the previous one. You can do this by typing the formula `=B3*B2` in cell **C3** and filling down.

This does not produce a list of Fibonacci numbers but if you look carefully you can see that they each have factors which are Fibonacci numbers.

Squares

Find all the squares of the Fibonacci numbers in your spreadsheet. You can do this by adding formulas in the next available column in your spreadsheet. In the cell **D2**, type the formula `= A2*A2`. This will find the square of the first Fibonacci number and you can then fill down this formula to find the squares of all the others.

Now use your Excel spreadsheet to make running totals of the squares. You can do this by following these steps:

- Type in **1** in cell **D2**. This is simply the square of the first Fibonacci number
- Now type in the formula `=C3+D2` in cell **D3** and fill down.

Compare the results of this with the results you obtained by multiplying each Fibonacci number by the previous one. What is the connection? Suggest a rule for finding the sum of the first **n** Fibonacci numbers.

More multiplying

Try adding formulas that will multiply the 1st Fibonacci number by the 3rd, the 2nd by the 4th, the 3rd by the 5th and so on. Compare your results with your list of the squares of the Fibonacci numbers. What is the connection?

Try adding formulas that will multiply a Fibonacci number by the 3rd, the 2nd by the 4th, the 3rd by the 5th and so on. Compare your results with your list of the squares of the Fibonacci numbers. What is the connection?

Finding the Golden Ratio



The line above, the ratio $a+b : a =$ the ratio $a : b$. This provides an example of the Golden Ratio which is the ratio $\frac{1 + \sqrt{5}}{2} : 1$ or approximately 1.618033989:1

Amazingly, it can be shown that the ratio of successive Fibonacci numbers gets closer and closer to the Golden Ratio. Follow these steps to observe this happening using your Excel spreadsheet.

- In cell **C3** enter the formula = **B3/B2**
- Fill down the formula in cell **C3**

Notice how the results appear to get closer and closer to the Golden Ratio.

Primes

Which Fibonacci numbers are prime? (Warning: This could be your life's work!)

Drawing patterns with Fibonacci squares

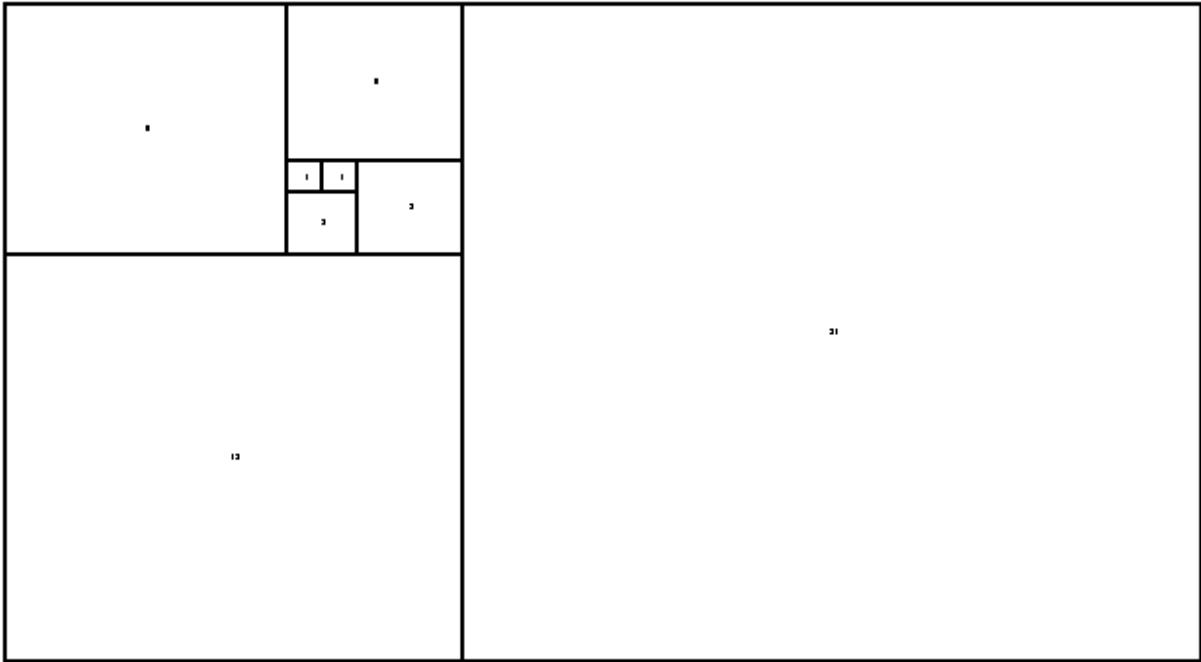
Interesting patterns can be drawn with Fibonacci squares. First of all you will need to make the cells in your spreadsheet reasonably square in shape. Follow these steps to do this:

- Start a fresh spreadsheet by clicking on **Sheet2** at the bottom of the page
- Select the first 40 columns by clicking on the **A** at the top left of the spreadsheet and dragging across to **AN**
- On the **Home** tab, in the **Cells** group, select **Format Column Width** and enter **8.11** for the column width
- Select the first 40 rows by clicking on the **1** at the top left of the spreadsheet and dragging down to row 40
- On the **Home** tab, in the **Cells** group, select **Format Row Height** and enter **42.00** for the row height

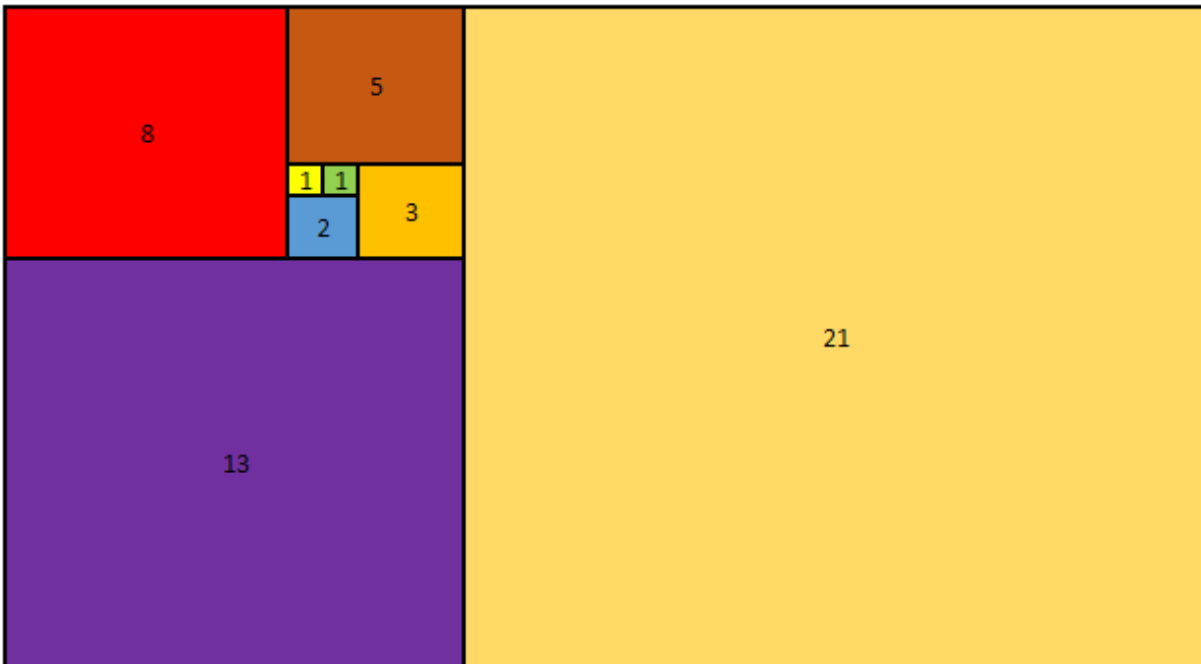
Now that you have a grid to work with, follow these steps to start making Fibonacci squares into a pattern:

- Enter the 1st Fibonacci number, **1** into cell **N15**
- Click on cell **N15**
- On the **Home** tab, in the **Font** group, select **Thick Box Border**
- On the **Home** tab, in the **Alignment** group, select **Middle Align**
- Enter the 2nd Fibonacci number, **1** into cell **O15**
- Format cell **O15** in the same way that you did for cell **N15**
- Now enter the 3rd Fibonacci number, **2** in cell **N16**
- Select cells **N16**, **O16**, **N17**, and **O17**
- On the **Home** tab, in the **Font** group, select **Thick Box Border**
- On the **Home** tab, in the **Alignment** group, select **Merge & Centre** and also select **Middle Align**

If you repeat the process of adding squares, then you will be able to construct an interesting pattern as shown on the next page.



It will be helpful to increase the font size of the numbers and fill the squares with different colours



Notice the way that the Fibonacci numbers are forming a spiral shape.